

# EFFECTIVE MODELS OF THE ELECTROWEAK PHASE TRANSITION\*

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The consecutive integration over the distinct mass scales  $\mathcal{O}(T), \mathcal{O}(gT)$  leads to a hierarchy of effective models for the electroweak phase transition. Different techniques for the realisation of such strategy are reviewed. Advantages and difficulties resulting from the use of reduced models are discussed.

## 1. Introduction

Infrared improved perturbative treatments of the electroweak phase transition<sup>1,2,3,4,5</sup> have reinforced the pioneering suggestion of Kirzhnits and Linde<sup>6</sup>, that "massless" finite temperature magnetic fluctuations might drive this transition into the first order regime. The effect of the out-of-equilibrium state associated with a discontinuous transition might be essential for understanding the possible origin and the survival of a cosmological B–L asymmetry<sup>7,8</sup>.

The question of existence of a characteristic mass scale of finite temperature non-Abelian magnetic fluctuations has been discussed extensively in the past decade, by means of numerical<sup>9,10,11</sup> and semi-classical<sup>12</sup> techniques, and with help of self-consistent coupled Schwinger–Dyson equations<sup>2,3,13,14</sup>. Also, it has been argued that several finite temperature quantities (e.g. electric screening mass, etc.) would be sensitive to the existence of a non-zero magnetic scale<sup>15</sup>. All approximate studies conjecture the magnetic scale to be of  $\mathcal{O}(g^2T)$ , ( $g$  being the gauge coupling). More generally, it is expected that phenomena which are perturbatively not accessible in finite temperature Higgs models would occur at this scale.

Therefore with no hesitation one can integrate out of the partition function of the finite temperature electroweak theory all non-static Matsubara modes, since they are characterised by the high momentum scale  $2\pi T$ . This is the background for the application of the "conventional" reduction program, which has been realised for the Higgs-systems to date at 1-loop level<sup>16,17,18</sup>. In the course of the integration the static parts of the electric components of the vector fields,  $A_0$  and of the Higgs-doublet receive thermal mass contributions  $\mathcal{O}(gT)$ .

In this way the  $A_0$ -field (at least in the weak coupling limit) defines a new scale which is still distinctly bigger than the magnetic one, therefore it can also be integrated out of the theory. It is important to emphasize that the elimination of the

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static  $A_0$  is justifiable only in the present hierarchical approach. The experimental value of  $g \sim 2/3$ , however might raise doubts about the quantitative accuracy of another 1-loop approximation<sup>17,19</sup>, even though in terms of the original theory it corresponds to an infinite resummation. More accurate integration schemes (for instance, Renormalisation Group improved 1-loop integration<sup>20</sup>) might be invoked in order to estimate the systematic errors introduced by simpler approximations.

A similar integration over the Higgs fields is not advisable, especially in the temperature range of the phase transition, where the thermal contribution tends to compensate the wrong sign squared mass, defining the theory at  $T = 0$ .

In section 2, I summarize 1-loop results for reduced theories of the electroweak phase transition. In section 3 the strategy of non-perturbative (lattice MC) investigations of the effective theories will be outlined. Its application will be illustrated on the example of a pure scalar (order-parameter) effective theory of the electroweak phase transition<sup>21</sup>. The effects of non-renormalisable and of non-local operators, appearing when more accurate mapping of the full theory on a 3 dimensional effective model is attempted, will be discussed in section 4 on the example of the finite temperature  $O(N)$  symmetric scalar model<sup>22</sup>. In particular, I shall compare the strategy of consecutive integrations to the "matching" program of Braaten and Nieto<sup>23</sup>.

## 2. Hierarchy of effective theories at 1-loop level

The object of study in most investigations is the Euclidean version of the  $SU(2)$  Higgs model:

$$L[A_\mu, \phi] = \frac{1}{4} F_{mn}^a F_{mn}^a + \frac{1}{2} (D_m \phi)^\dagger (D_m \phi) + \frac{1}{2} m^2 \phi^\dagger \phi + \frac{1}{24} (\phi^\dagger \phi)^2 + L_{c.t.}^{4D}, \quad (1)$$

with  $F_{mn}^a = \partial_m A_n^a - \partial_n A_m^a + g \epsilon^{abc} A_m^b A_n^c$ ,  $D_m \phi = (\partial_m + ig \tau^a A_m^a / 2) \phi$ .  $A_m^a$  is the 4-dimensional vector field transforming as a triplet under the  $SU(2)$  gauge group, while  $\phi$  is a complex Higgs-doublet.  $L_{c.t.}^{4D}$  refers to the temperature independent counterterms absorbing ultraviolet singularities of the 4-dimensional theory. One expects that once controlling the phase transition of this simplified system one will be able to handle the complete  $SU(2) \times U(1)$  theory. The inclusion of chiral fermions, however, is yet posing insurmountable difficulties to lattice studies of the full 4-dimensional finite temperature model. In this respect, the integration over all fermionic modes (characterised by the mass scale  $\pi T$ ) is the only practical solution.

### 2.1. Integration of non-static modes

Complete 1-loop integration has been performed in the background of static  $A_i(x), A_0(x), \phi(x)$  fields in general covariant gauge<sup>18</sup> and also in Landau gauge<sup>19</sup>. With appropriate (gauge parameter dependent) field-, mass- and coupling constant renormalisations one arrives at the following gauge independent effective (reduced)

system in 3 dimensions:

$$\begin{aligned}
L_{eff}^I = & \frac{1}{4}F_{ij}^a F_{ij}^a + \frac{1}{2}(D_i^{adj} A_0)^a (D_i^{adj} A_0)^a + \frac{1}{2}(D_i \phi)^\dagger (D_i \phi) \\
& + \frac{1}{2}m_3^2 \phi^\dagger \phi + \frac{1}{2}m_D^2(T)(A_0^a)^2 \\
& + \frac{1}{2}g_3^2(A_0^a)^2 \phi^\dagger \phi + \frac{1}{24}\lambda_3(\phi^\dagger \phi)^2 + \frac{17g_3^4}{192\pi^2 T}[(A_0^a)^2]^2 + \text{higher dim. op's},
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
g_3^2 &= g_R^2 T, \quad \lambda_3 = \lambda_R T, \quad m_D^2(T) = \frac{5}{6}g_3^2 T, \\
m_3^2 &= m_R^2 + (\frac{3}{16}g_3^2 + \frac{1}{12}\lambda)T,
\end{aligned} \tag{3}$$

$((D_i^{adj} A_0)^a = (\partial_i \delta^{ac} + g_3 \epsilon^{abc} A_i^b) A_0^c)$ . The quantities with index "R" above refer to couplings renormalised in 4 dimensional sense (this index will be omitted below). The omission of higher dimensional operators from the effective action will be discussed in section 4.

Counterterms of this 3 dimensional system cannot be chosen freely: the procedure of the "projection" of the full system onto the static variables should induce those singularities of the action which are necessary for ensuring the finiteness of the static effective theory. Even those divergences which would arise if higher dimensional (usually called non-renormalisable) operators would be retained in the effective action should be canceled by appropriate induced counterterms. In principle, therefore, it presents no particular interest to distinguish between approximate versions of the effective theories which are renormalisable on their own in 3-dimensional sense and those which are not. At 1-loop level of the reduction, however, only mass counterterms are produced ( with a sharp momentum cut-off):

$$L_{c.t.}^I = -\frac{1}{2}\phi^\dagger \phi \left( \frac{9}{4}g_3^2 + \lambda_3 \right) \frac{\Lambda}{2\pi^2} - \frac{1}{2}(A_0^a)^2 5g_3^2 \frac{\Lambda}{2\pi^2}. \tag{4}$$

## 2.2. Integration of the $SU(2)$ triplet field $A_0^a$

Further integration over fields characterised by the scale  $\mathcal{O}(gT)$  has been proposed in <sup>19,17</sup>. 1-loop integration can easily be performed (there is no need for fixing any gauge) resulting in a variant of the 3D Higgs model:

$$\begin{aligned}
L_{eff}^{II} = & \frac{1}{4\mu}F_{ij}^a F_{ij}^a + \frac{1}{2}(D_i \phi)^\dagger (D_i \phi) + \frac{1}{2}m_3^2 \phi^\dagger \phi + \frac{1}{24}\lambda_3(\phi^\dagger \phi)^2 \\
& - \frac{1}{4\pi}(m_D^2 + \frac{1}{4}g_3^2 \phi^\dagger \phi)^{3/2} + L_{c.t.}^{II}, \\
\mu = & 1 + \frac{g_3^2}{24\pi(m_D^2 + \frac{1}{4}g_3^2 \phi^\dagger \phi)^{1/2}}, \quad L_{c.t.}^{II} = -\frac{1}{2}\phi^\dagger \phi \left( \frac{3}{2}g_3^2 + \lambda_3 \right) \frac{\Lambda}{2\pi^2}.
\end{aligned} \tag{5}$$

Consistent omission of higher dimensional operators requires setting in the expression of the "magnetic susceptibility" either  $\phi = 0$  or  $\phi = \phi_{min}$ , while the non-analytic piece

of the potential should be expanded into power series of  $\phi^\dagger\phi$  up to the  $\mathcal{O}((\phi^\dagger\phi)^2)$  term. The final 3-dimensional Higgs model is characterised by the modified couplings

$$\bar{m}_3^2 = m_3^2 - \frac{3g^3}{16\pi}\sqrt{\frac{5}{6}}T^2, \quad \bar{\lambda}_3 = \lambda_3 - \frac{27g^3}{160\pi}\sqrt{\frac{5}{6}}T, \quad \bar{g}_3^2 = g_3^2 - \frac{g_3^4}{24\pi m_D}. \quad (6)$$

(In the present form of the effective couplings, expansion around  $\phi^\dagger\phi = 0$  is assumed.)

An attempt has been made to go beyond the 1-loop accuracy in the integration of  $A_0$  <sup>20</sup>. The evolution of the joint potential of  $A_0$  and  $\phi$  has been followed as the upper limit on the momenta of the Fourier expansion of  $A_0$  has been lowered, with help of an "exact" renormalisation group equation. Technically, the dependence of the potential on  $A_0$  has been made formally quadratic with the introduction of an auxilliary field  $\chi$ . Assuming constant (low frequency) background for  $A_0$ ,  $\phi$ ,  $\chi$  one can integrate over the infinitesimal momentum layer of  $A_0$ . In order to determine the functional dependence of  $\chi$  on  $A_0$  and  $\phi$ , the resulting expression of the potential energy term of the action is extremised in  $\chi$ . The following differential change of the potential energy is found:

$$k \frac{\partial U_k(\phi, A_0)}{\partial k} = -\frac{3}{4\pi^2} k^3 \log(k^2 + m_D^2(T, k) + \chi[A_0, \phi]_k + g_3^2(T, k)\phi^\dagger\phi), \quad (7)$$

where  $m_D^2(T, k)$ ,  $g_3^2(T, k)$  are the coefficients of  $A_0^2/2$  and of  $A_0^2\phi^\dagger\phi/2$ , at the actual upper momentum scale  $k$ , respectively. Their expressions appearing in (3) are the initial values for the integration of (7), imposed at  $k = \Lambda$ .

The couplings of the effective Higgs theory are obtained when one ends the integration of (7) at  $k = 0$ . Assuming that  $m_D^2(T, k)$  deviates from  $m_D^2(T, \Lambda)$  only in higher orders of  $g_3$ , and  $\phi$  fluctuates on scales smaller than  $\mathcal{O}(T)$ , one can justify the expansion of both sides of (7) into a polynomial expression of the fields. Truncating the infinite coupled set of differential equations at dimension 4 operators, one finds the following approximate solution for the correction of (3):

$$\begin{aligned} \bar{m}_3^2 - m_3^2 &= -\frac{3g^3}{16\pi}\sqrt{\frac{5}{6}}T^2(1 + \sqrt{\frac{6}{5}}\frac{5g}{\pi^2})(1 + \sqrt{\frac{6}{5}}\frac{17g^3}{128\pi^3})^{-1}, \\ \bar{\lambda}_3 - \lambda_3 &= -\frac{27g^3}{160\pi}\sqrt{\frac{5}{6}}T(1 + \sqrt{\frac{6}{5}}\frac{17g^3}{128\pi^3})^{-1}, \\ L_{c.t.}^{II} &= -\frac{1}{2}\phi^\dagger\phi(\frac{3}{2}g^2 + \lambda - \sqrt{\frac{5}{6}}\frac{15}{32\pi}g^3)\frac{\Lambda T}{2\pi^2}. \end{aligned} \quad (8)$$

The variation seen in  $\bar{m}_3^2 - m_3^2$  and in  $L_{c.t.}^{II}$  arising from the 3-dimensional running of the couplings, driven by the integration over  $A_0$ , is about 15% (for the realistic range of the gauge coupling values: 0.5-0.7) relative to the result of the 1-loop integration over  $A_0$ .

### 3. Lattice investigation of the effective 3D Higgs model

The lattice Higgs action with general scalar self-interaction potential  $V_4$  is written

in terms of field variables, appropriately scaled by the lattice spacing, as follows:

$$S[U_{xi}, \psi_x] = \sum_x [\frac{1}{2\kappa} \psi_x^\dagger \psi_x - \frac{1}{2} \sum_i (\psi_x^\dagger U_{xi} \psi_{x+i} + \psi_{x+i}^\dagger U_{xi}^\dagger \psi_x)] + \sum_x V_4(\psi_x^\dagger \psi_x) + S_{gauge} \quad (9)$$

The hopping parameter  $\kappa$  is related to the quadratic part of the bare action through the equality:

$$\frac{1}{2\kappa} = \frac{1}{2} m_R^2 a^2 + \frac{1}{2} (\frac{3}{16} g^2 + \frac{1}{12} \lambda - \frac{3g^2 m_D}{16\pi T}) \Theta^2 - \Theta \Sigma(N^3) \frac{f_{1m}}{2} + 3, \quad (10)$$

with  $\Theta \equiv aT$ ,  $f_{1m} = 3g^2/2 + \lambda$ ,  $\Sigma(N^3) = \sum_n (4N^3 \sin^2(\pi n_i/N))^{-1}$ ,  $a$  is the lattice spacing,  $N$  is the lattice size.

For any finite value of  $\Theta$  one can study the phase transition occurring in (9) at  $\kappa = \kappa_c(\Theta)$ . Under the assumption that the effective action is correctly representing the finite temperature system up to arbitrary high spatial momenta one takes the limit  $a \rightarrow 0$  and determines the nontrivial limiting quantity:

$$Z_c = \lim_{\Theta \rightarrow 0} (\frac{1}{2\kappa_c(\Theta)} - 3 + \Theta \Sigma(N^3) \frac{f_{1m}}{2}) \frac{1}{\Theta^2}. \quad (11)$$

Using  $Z_c$  in (10) one finds the physical value of the transition temperature in proportion to the renormalised mass parameter  $m_R^2$  (or the Higgs mass):

$$\frac{m_R^2}{2T_c^2} = Z_c - \frac{1}{2} (\frac{3}{16} g^2 + \frac{1}{12} \lambda - \frac{3g^2 m_D}{16\pi T}). \quad (12)$$

Relation (11) has been analysed very carefully for a model of the electroweak phase transition with all variables integrated out on 1-loop level, but the Higgs-dublet <sup>21</sup>. Careful quantitative analysis has shown (with the Higgs mass chosen around 35 GeV) that  $\kappa_c^{-1}$  actually follows quadratic dependence on  $\Theta$  in the interval  $\Theta \in 0.1 - 1.0$ . Detailed finite size scaling analysis of the lattice data for  $\Theta \in 1. - 3.$  have proven that the transition is discontinuous. Monte Carlo estimates of the latent heat, order parameter jump, etc. are easy to translate into physical units once the transition temperature is known. However, the results for the order parameter discontinuity, the latent heat, and especially for the interface tension were systematically underestimating the results obtained in simulations of more complete representations of the electroweak theory <sup>24,25</sup>. One reason for this is certainly the oversimplified perturbative treatment of the magnetic vector fluctuations (though magnetic screening has been accounted for by introducing the corresponding screening length into the scalar model in analogy to the Debye mass, cf. (5), by hand). Another question is whether it is correct to expect that the *exact* continuum limit of the cut-off effective theory should provide the most faithful representation of the original finite temperature

theory.

#### 4. More accurate reduction: non-renormalisable and non-local operators in the effective action?

Higher dimensional operators appear already in 1-loop reduction upon expanding the fluctuation determinant in higher powers of the background fields. In <sup>17</sup> the strength of all non-derivative dimension 6 operators has been extracted and small numerical coefficients gave argument for their consistent omission. Even though, if these operators are included into the solution of the effective model they contribute a linearly divergent piece to the couplings of dimension 4 operators (we give the expression in lattice regularisation):

$$\begin{aligned} \Sigma \frac{1}{\Theta} \{ & -[(A_0^a)^2]^2 \frac{\zeta(3)g^6}{16\pi^4} - (A_0^a)^2 \phi^\dagger \phi \frac{\zeta(3)g^2}{2048\pi^4} [\frac{967}{2}g^4 + \frac{47}{3}\lambda g^2 + \frac{5}{3}\lambda^2] - \\ & -(\phi^\dagger \phi)^2 \frac{\zeta(3)}{1024\pi^4} [\frac{255}{16}g^6 + \frac{65}{2}\lambda g^4 + \frac{23}{3}\lambda^2 g^2 - \frac{40}{9}\lambda^3] \}. \end{aligned} \quad (13)$$

Since these operators are of  $\mathcal{O}(g^6, \lambda g^4, \dots)$ , the corresponding "counterterms" will be induced at 3-loop level of the reduction. If one does not include into the expected  $\Theta$ -dependence of the corresponding bare couplings these contributions, a theoretical error is introduced into the reduced description. In the  $\Theta$  region where the terms with inverse  $\Theta$ -dependence are negligible the error is negligible. Clearly this requirements (one for the coupling of each dimension 4 operator) sets a lower limit to the variation of  $\Theta$ . When one substitutes the usual numerical range for  $g$  and  $\lambda$  it turns out that these lower limits for the grain size are  $\mathcal{O}(10^{-2})(1/T)$ . This makes the preceding discussion only of conceptual interest, since these limits represent a warning that one should not take for any of the approximate reduced models the strict continuum limit.

Effects of two-loop level reduction in the 3 dimensional representation of finite T field theories has been discussed on the example of the N-component scalar theory <sup>22</sup>:

$$S = \int_0^\beta d\tau \int d^3x [\frac{1}{2}(\partial_\mu \phi_\alpha)^2 + \frac{1}{2}m^2 \phi_\alpha^2 + \frac{1}{24}\lambda(\phi_\alpha^2)^2]. \quad (14)$$

The coefficients  $m_3^2(T)$ ,  $\lambda_3(T)$  have been explicitly determined:

$$m_3^2 = m^2 + (\frac{1}{24}\lambda - 0.001355\lambda^2)\frac{N+2}{3}T^2, \quad \lambda_3 = \lambda T. \quad (15)$$

Counterterm has been induced only to the mass of the 3 dimensional theory:

$$\delta m_3^2 = -\frac{N+2}{3}[\frac{\lambda}{4\pi^2}(1 - 0.048277\lambda)\Lambda T + \frac{\lambda^3}{32\pi^4}\Lambda T \log \frac{\Lambda}{T} + \frac{\lambda^2}{48\pi^2}T^2 \log \frac{\Lambda}{T}] \quad (16)$$

(again cut-off regularisation has been used).

A puzzling observation is made when one calculates at 2-loop level the effective potential of the effective theory with parameters taken from (15). The divergent part of the potential turns out to be

$$U_{div}(\phi_0) = \frac{1}{2}\phi_0^2 \frac{N+2}{3} \left[ \frac{\lambda}{4\pi^2} \Lambda T - \frac{\lambda^2}{192\pi^2} T^2 \log \frac{\Lambda^2}{\mu_3^2} \right]. \quad (17)$$

This result agrees with the scalar part of the 2-loop calculations done for the 3 dimensional SU(2) Higgs model by several authors<sup>19,26,27</sup>. There is no doubt that one encounters a mismatch between all 3 types of divergences occurring in the 2-loop mass counterterm and the divergent piece of the effective potential they should cancel. One should interpret this as a signal that the representation of the finite T theory with an effective action containing local operators up to dimension 4 cannot be correct at 2-loop level. By the previous discussion also the possible impact of higher dimensional local operators can be ruled out, since they are of higher order in  $\lambda$ .

The resolution proposed in<sup>22</sup> is to include also a momentum dependent 4-point vertex into the effective theory. This operator is constructed to reproduce those contributions to the effective potential of the full finite temperature theory, which diagrammatically contain both static and non-static internal lines. Such situation occurs first at 2-loop level, what explains why at 1-loop no mismatch has been observed. For the N-component scalar model the non-local term was found to be of the following form:

$$\begin{aligned} L_{non.loc.}^{3D} &= \frac{\lambda^2}{128\pi^2} \left( \frac{N+4}{9} O_1 + \frac{4}{9} O_2 \right), \\ O_1 &= \phi_\alpha^2 \Omega(i\partial) \phi_\beta^2, \quad O_2 = \phi_\alpha \phi_\beta \Omega(i\partial) \phi_\alpha \phi_\beta, \\ \Omega(k) &= \frac{\pi^2 T k}{T^2 + k^2} + \frac{1}{2} \log(1 + \frac{k^2}{4C^2 T^2}) + F(\frac{k}{\Lambda}), \end{aligned} \quad (18)$$

with  $C = 2\pi \exp(1 - \gamma_E)$  and  $F(x) = (2/x - 1) \log(1 - x/2) + 1$ .

Since  $L_{non.loc.}^{3D}$  is  $\mathcal{O}(\lambda^2)$  for the solution of the effective theory to  $\mathcal{O}(\lambda^2)$  it is sufficient to calculate its contribution at 1-loop. Then one can check explicitly that its divergent contribution exactly fills the gap between (17) and (16). This means that at least to two-loop accuracy there is consistency between the full theory and its reduced image up to arbitrarily high spatial momenta.

If one is not interested in momenta above the scale  $\mathcal{O}(T)$ , the effect of the non-local terms would be compressible into the couplings of a local effective  $\phi^4$ -type theory of the effective theory. Also the field variables of the two theories might differ, therefore one should introduce a field rescaling factor Z:

$$L_{eff,eff} = \frac{1}{2}(\partial_i \hat{\phi}_\alpha)^2 + \frac{1}{2} \hat{m}_3^2 \hat{\phi}_\alpha^2 + \frac{\hat{\lambda}}{24} (\hat{\phi}_\alpha^2)^2, \quad Z \phi_\alpha^2 = \hat{\phi}_\alpha^2. \quad (19)$$

For the determination of the couplings one can follow the matching strategy of<sup>23</sup> and require the equality of the 2- and 4-point functions calculated from (19) and from the

non-local theory at small spatial momenta. Systematically throwing away terms of  $\mathcal{O}(m_3^2/T^2)$  one finds, for instance:

$$\begin{aligned} Z &= 1 - \frac{(N+2)\lambda^2}{96\pi^2} \left( \frac{1}{6} \log \frac{T^2}{m_3^2} + \frac{1}{2} + \frac{1}{12\pi C} + \mathcal{O}\left(\frac{m_3^2}{T^2}, \lambda\right) \right), \\ \hat{m}_3^2 &= m_3^2(T) + \frac{N+2}{3} \lambda^2 T^2 \left( \frac{C}{32\pi^3} + \mathcal{O}\left(\frac{m_3^2}{T^2} \log \frac{T^2}{m_3^2}, \lambda\right) \right). \end{aligned} \quad (20)$$

Actually, the matching strategy of the last step has been applied directly to finding the coupling parameters of the local 3 dimensional Higgs model representation of the finite temperature SU(2) Higgs system<sup>19</sup>. The 2-loop effective potential of the effective theory has been computed. In this superrenormalisable theory only the mass needs renormalisation and the constant characterising its scale dependence has been determined by comparing the result to the 2-loop calculation of the effective potential in the full model. For the interpretation of the lattice investigations of this particular effective model the analogue of the relation (10) has been carefully determined to 2-loop accuracy<sup>28,29</sup>. In view of the above discussion it cannot be taken granted that the scaling behavior of the local effective theory, valid in that model for very large values of the cut-off, can be observed for  $aT \sim \mathcal{O}(1)$ , where this theory is expected to describe the original finite temperature theory well. The observation of a scaling window around  $\Theta \sim 1$  seems to be a rather non-trivial task. Even if it can be observed for a specific discretisation, one ought to check the compatibility of the results obtained with different discretisations (in order to check the absence of finite lattice spacing effects).

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